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Lower bound for the hardy constant for an arbitrary domain in R^n

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Abstract

© I.K. Shafigullin. 2017. In the paper we consider the conjecture by E.B. Davies on an uniform lower bound for the Hardy constant. We provide the known counterexamples rebutting this conjecture for the dimension 4 and higher. In the work we obtain non-zero lower bounds for the Hardy constants. These estimates are order sharp as $n \to +\infty$, where n is the space dimension. Moreover, these estimates are independent of the properties of the considered domains and are true for all domains not coinciding with the entire space. In the proof of the main theorem we reduce the multidimensional case to the one-dimensional case by choosing special classes of functions. As a result, the considered inequalities are reduced to the well-known Poincaré inequality.

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Keywords

Hardy constant, Hardy inequalities, Lower bounds, Variational inequalities

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